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# Prediction of capacity for moment redistribution in FRP-strengthened continuous RC T-beams

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## ABSTRACT

Due to the premature debonding of fiber-reinforced polymer (FRP) materials which results in a reduction in ductility, the problem of how to exploit moment redistribution (MR) in FRP-strengthened continuous reinforced concrete (RC) structures is still unresolved. To date, limited research has been conducted into MR in such structures, so that a reliable and rigorous solution for quantifying MR throughout the loading cycle remains elusive. This paper aims to quantify MR and predict the capacity at reasonable accuracy, to encourage the use of FRP for the strengthening of existing continuous RC structures. Experiments conducted on twelve continuous T-beams are reported, and the findings are discussed. Strengthening configuration and anchorage scheme are the main variables. A new analytical strategy is described for quantifying MR, and the analytical results are then validated against the experimental results. Both experimental and analytical results confirm that there is no reason to restrict MR into strengthened zones. More importantly, MR out of FRP-strengthened zones can indeed occur, provided that the FRP is sufficiently anchored, and reliable exploitation of this is now possible.

## Keywords:

Moment redistribution; continuous members; FRP strengthening; concrete T-beams; FRP anchorage

## 29    **Introduction**

30    To avoid the need for replacement or demolition of existing reinforced concrete (RC) structures, they are  
31    routinely strengthened using various materials and techniques. Research in the literature (Meier et al., 1993;  
32    Teng et al., 2001; ACI440-2, 2008) has demonstrated the effectiveness of fiber reinforced polymer (FRP)  
33    materials in extending the lifetime of existing RC structures. FRP strengthening of concrete members is known  
34    to be a rapid and cost-effective method of strengthening. Thus, FRP is currently used widely for the retrofit of  
35    RC structures.

36    Although FRP can considerably improve the strength capacity of an existing RC structure, previous research  
37    (Duthinh and Starnes, 2004; Oehlers, 2006; Yost et al., 2007) has shown that the ductility of RC structures can  
38    be reduced after strengthening. The two main reasons for this problem are the elastic nature of FRP which  
39    reduces overall curvature ductility of the original member, and the premature and brittle debonding of the FRP  
40    from the concrete surface which prevents the ultimate strength of the FRP from being achieved. As a result,  
41    the reduction in ductility is considered to affect substantially the degree of moment redistribution which can  
42    take place following the FRP strengthening of an existing continuous RC flexural member.

43    The required level of ductility for moment redistribution (MR) is unclear in FRP-strengthened continuous RC  
44    members, and there is a lack of sufficient research to demonstrate a precise level of ductility reduction after  
45    adding FRP. Therefore, the exploitation of MR in the design of FRP strengthening systems has been  
46    conservatively ignored or restricted by design codes and guides worldwide (e.g. ACI 440.2R, 2008; TR55,  
47    2012). This potentially compromises the safety of such strengthened structures under extreme loads since  
48    implication of the lower-bound theorem of plasticity can no longer be relied on for redistribution of load paths.  
49    In addition, it should be noted that if MR is ignored in an FRP-strengthened RC member which was originally  
50    designed assuming MR, the strengthened member must be necessarily analyzed using elastic equations.  
51    Consequently, great quantities of FRP must be added to the member because the fully-elastic situation must  
52    now be considered even for the original situation. Therefore, there is a pressing need to investigate fully how  
53    MR might be understood and exploited in the strengthening of continuous RC structures.

Potentially, it is difficult and complex to quantify the actual level of ductility, and to predict the capacity for MR when FRP is added to an RC member (Oehlers et al., 2004). Few research studies have experimentally or theoretically investigated redistribution of bending moments in FRP-strengthened RC structures. For example, El-Refaie et al. (2003) tested eleven two-span rectangular beams strengthened using externally bonded (EB) FRP sheet. They found that the quantity and arrangement of the internal steel reinforcement, as well as the quantity of the FRP applied, are the most important factors influencing MR. They recommended that an anchorage system for the FRP should be provided to minimize the risk of premature FRP peeling. They showed significant MR is possible out of strengthened zones, with their particular tests demonstrating up to 35% MR.

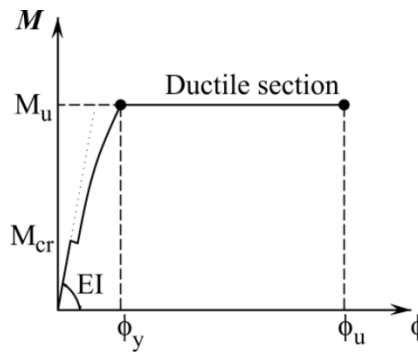
In a theoretical study, Oehlers et al. (2004) proposed two approaches, called the 'Flexural rigidity approach' and the 'Plastic hinge approach', to quantify redistribution of bending moments. The two approaches were based on 'stiffness variation' and a 'hinge zone', respectively. They also tested four two-span rectangular slab-shaped concrete beams to measure any possible MR. The beams were strengthened only in the negative zone (over the interior support), using EB CFRP plates. MR up to 35% was found in their particular tests, depending on the arrangement of the internal-steel-reinforcement adopted.

Limited studies have also been conducted by other researchers (Silva and Ibell, 2008; Aiello and Ombres, 2011; Dalfré and Barros, 2011; Breveglieri et al., 2012; Santos et al., 2013; Lou et al., 2015), which show that MR can occur to a significant extent after FRP strengthening, provided that an appropriate strengthening configuration is adopted.

This paper initially presents the findings of a set of experiments, aiming at quantifying MR in FRP-strengthened continuous RC T-beams. Various strengthening configurations and techniques were adopted to evaluate the effect on MR. Twelve two-span RC T-beams were tested in two groups. In addition, the FRP was anchored mechanically in some of the specimens to understand the potential influence of anchorage on the degree of MR. It must be noted that quantification of the effectiveness of the anchorage system itself is not the purpose of this paper. The experimental results are then compared with the analytical results obtained from a novel analytical model developed by the authors (Tajaddini et al., 2013; Tajaddini, 2015). The analytical results is used to quantify the full potential capacity of the tested members for MR, if the FRP were not to debond prior to concrete crushing or FRP rupture.

## Moment redistribution (MR)

The implication of MR in statically indeterminate structures has been described in the literature (Bondy, K.B., 2003; Oehlers et al., 2010; Bagge et al., 2014) through simple examples. RC structures are designed so that they resist external actions elastically within the serviceability load range. Beyond this range, if one (or more) section of the structure reaches its moment capacity, the section will rotate at a constant bending moment, forming a plastic hinge provided that the section has sufficient ductility. As shown in Fig. 1, an idealized elastic-plastic relationship between curvature and bending moment is assumed in an unstrengthened ductile section, in which  $M_{cr}$  is the bending moment at first cracking,  $M_u$  is the ultimate moment capacity,  $\phi_y$  is the curvature at steel yielding,  $\phi_u$  is the ultimate curvature, and  $EI$  is the uncracked flexural stiffness. Now, as the applied load is further increased, the critical point (plastic hinge location) will redistribute the extra bending moment to other parts of the structure to accommodate the increase in loading.



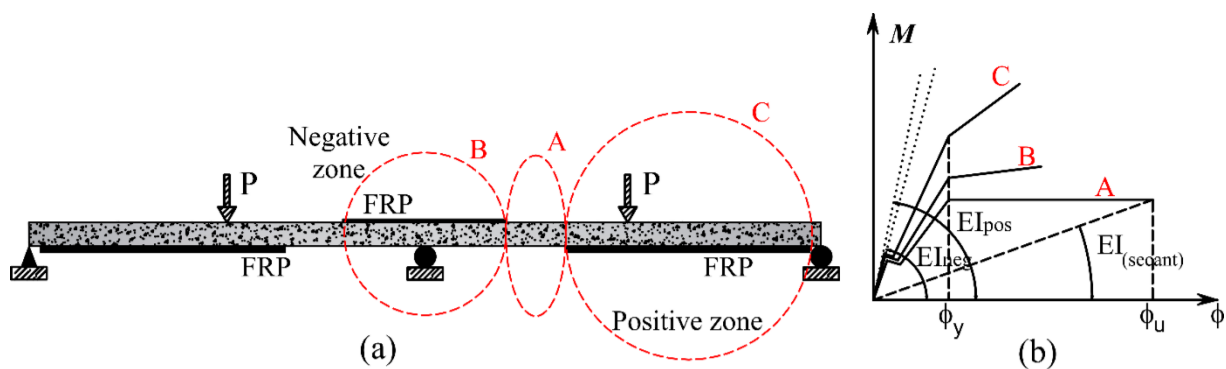
**Fig. 1.** An idealized elastic-plastic Moment-Curvature relationship in a ductile RC section

The redistribution of bending moment continues, and plastic hinges are formed successively in the structure, until a failure mechanism is formed and the structure collapses. Through this process, the structure withstands extra applied loads after yielding of the first section until the structure collapses ultimately. In the case of sufficient ductility, the initial elastic bending moment diagram can be significantly different from the final redistributed bending moment diagram at ultimate failure. Therefore, the ratio of the negative bending moment to positive bending moment does not remain constant. As described by El-Refaie et al. (2003), the amount of MR is calculated at each applied load increment (up to failure) using the following equation:

$$MR (\%) = 100 \times \left( 1 - \frac{M_{redistributed}}{M_{elastic}} \right) \quad (1)$$

101 where  $M_{redistributed}$  is the redistributed bending moment at a critical location at the applied load, and  $M_{elastic}$  is  
 102 the theoretical elastic bending moment determined from elastic analysis at the same location, assuming an  
 103 initial uncracked elastic flexural stiffness.

104 MR becomes a complex problem when FRP is added to a continuous RC beam. As illustrated in Fig. 2, an  
 105 FRP-strengthened continuous RC beam might have various zones which can be unstrengthened (e.g. Zone A),  
 106 lightly strengthened (e.g. Zone B), or heavily strengthened (e.g. Zone C). Over the loading cycle, each zone  
 107 experiences a specific level of stiffness variation which is different from other zones.



**Fig. 2.** (a) Schematic image of a continuous FRP-strengthened beam; and (b) Moment-Curvature relationships for different zones of the beam

108  
 109 As illustrated schematically in Fig. 2(b), a lack of horizontal plastic plateau in the Moment-Curvature  
 110 relationship of the FRP-strengthened zones (zones B and C) prevents plastic hinges forming in the strengthened  
 111 zones. This is because the FRP resists the applied load linearly until failure, even if the steel reinforcement  
 112 yields before FRP failure. The complexity of quantifying MR becomes greater if various amounts of FRP are  
 113 added to different parts of a concrete member. Applying various strengthening configurations and techniques  
 114 with different anchoring schemes affects the flexural behavior and failure mode of the strengthened member.  
 115 All these indicate that a comprehensive investigation of the problem is still required.

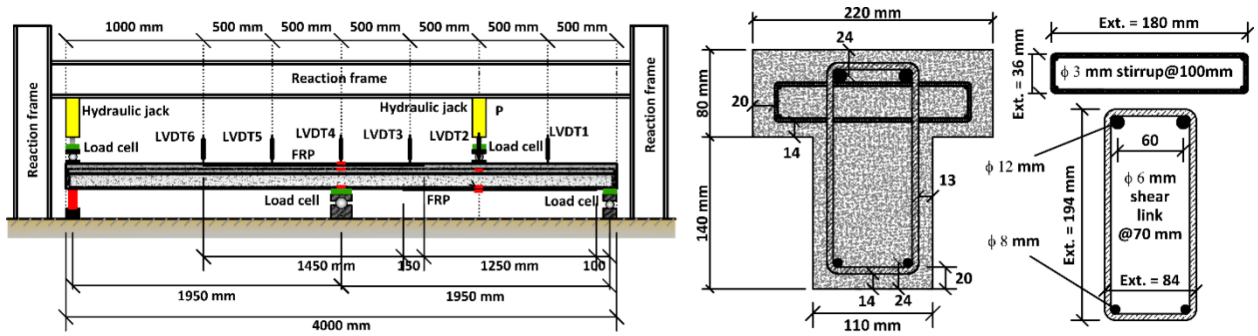
## 116 Experimental study

### 117 Test aim and program

118 A set of experiments were designed to examine the effect of FRP strengthening on the level of MR in  
 119 continuous RC flexural members. The test aims included:

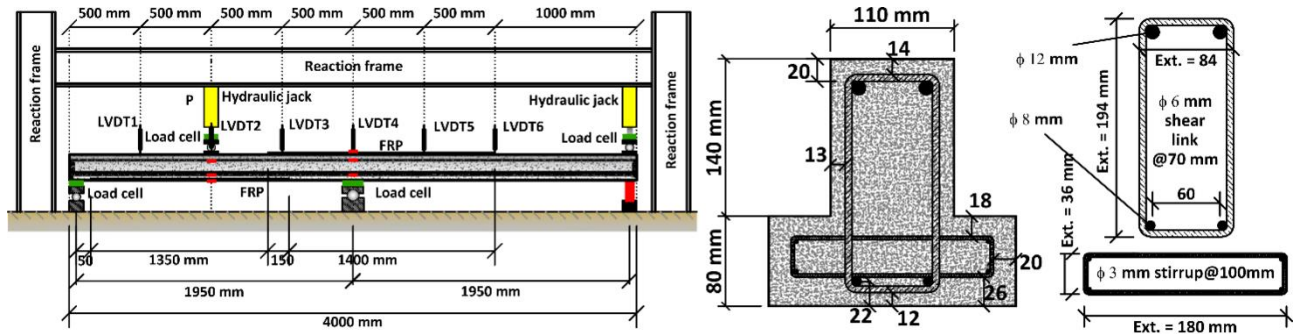
- To experimentally investigate MR in RC continuous T-beams when strengthened using FRP, as standalone rectangular beams are rarely found in reality;
- To examine the influence of various strengthening techniques and configurations on MR;
- To verify the new analytical model developed by the authors previously for quantifying MR;
- To understand the effect of FRP anchorage on the level of MR.

Test specimens were two-span and loaded at one of the mid-span points, using a concentrated load. This specific load arrangement caused the beam to only have one positive zone and one negative zone along its length. Therefore, this means that MR could only occur from one zone to the other (either from the positive zone to the negative zone or vice versa), ensuring MR could be easily tracked and quantified while still allowing different strengthening strategies to be explored. In addition, this asymmetrical arrangement of loading allowed the analytical model to be verified in a general sense as, for example, in the numerical procedure it would not matter whether rotation was zero or not at the position of central support. Twelve T-beams were designed and cast in two groups of T and U. Group T included six test specimens positioned in the usual upright T configuration, as shown in Fig. 3. Group U included six test specimens positioned upside-down so that MR could be studied comprehensively into and out of asymmetric sections. The main variables included configuration of the FRP strengthening and anchorage. The soffit of the positive zone of the specimens, under the load position, was strengthened (where applicable) using the externally bonded (EB) FRP plate, while the negative zone, over the central support, was strengthened (where applicable) using near surface mounted (NSM) FRP tape embedded into the surface of the flange. The NSM technique was used as a column would be in the way in reality, and it would be impossible to use the EB FRP plate or sheet for strengthening of such zones. Fig. 3 illustrates a schematic image of the geometry, loading arrangement and cross-section of the beams in group T. All specimens were designed such that they could exhibit up to 30% MR before FRP strengthening, as recommended by design guidelines such as BS 8110-1 (2005), AS 3600 (2009), and CSA A23.3 (2014) for conventional RC members. Note that, ACI-318 (2014) limits MR to 20% in such members, but the reality is that more MR can be achieved.



**Fig. 3. Geometry, load arrangement and cross-section of the T-beams in group T**

Since the positive and negative bending moments were so different over the loading cycle due to the specific loading arrangement adopted, MR was only possible in one direction (i.e. from the positive zone to the negative zone). Therefore, as the positive zone was only strengthened with the EB technique, it would not be possible to quantify MR out of the negative zone which was strengthened with the NSM technique. To solve this problem and to examine the effectiveness of the NSM technique in redistribution of bending moment, the “upside-down” Group U was designed in order to quantify experimentally bending moment redistributed out of NSM-strengthened zones.



**Fig. 4. Geometry, load arrangement and cross-section of the T-beams in group U**

The overall length of the specimens was 4000 mm, and each span was 1950 mm long. A single concentrated load was applied at a distance of 1000 mm from the central support. Each beam had a 220-mm flange width, 220-mm height, 110-mm web width, and 80-mm flange depth. The beams were internally reinforced using two longitudinal 12-mm diameter steel bars at the top, and two longitudinal 8-mm diameter steel bars at the bottom of the section. These steel quantities were chosen to encourage high levels of potential MR 6mm-diameter stirrups were used in the web, spaced at 70-mm centers, to prevent shear failure. The flange was also reinforced against shear using 3mm-diameter stirrups, spaced at 100 mm centers.



161 Deflections of the test specimens were recorded continuously during the testing, using six Linear Variable  
162 Differential Transformers (LVDTs) placed on top of the beams (as shown in Figs. 3 and 4). Values of the  
163 applied load and support reactions were also recorded over the loading cycle, using digital load cells.  
164 Therefore, bending moments in both critical positive and negative zones could easily be calculated. Electrical  
165 resistance strain gauges (shown as small solid blocks in Figs. 3 and 4) were installed on the tension and  
166 compression steel reinforcement and on the FRP in both positive and negative zones to record strains, and to  
167 monitor the flexural softening of both zones during loading. A hydraulic jack was used on top of the exterior  
168 support in the unloaded (right-hand) span to prevent it moving upward, and a locked-off jack was also used  
169 below the specimens for ease of adjustment.

170 Table 1 summarizes specifications of the specimens in the two groups. Different strengthening configurations  
171 were adopted for the experiments to assess the degree of bending moment which could be redistributed into  
172 and out of the strengthened zones. One specimen was used as the control specimen in each group (i.e. T1 and  
173 U1). Beams T2 and T3 were strengthened only in the positive zone using EB carbon FRP plate. Beam T4 was  
174 strengthened only in the negative zone using NSM carbon tape. Beams T5 and T6 were strengthened in both  
175 the positive and negative zones using EB carbon plate and NSM tape, respectively. Beam U2 was strengthened  
176 only in the positive zone using NSM carbon tape. Beams U3 and U4 were strengthened only in the negative  
177 zone using EB carbon FRP plate. Beams U5 and U6 were strengthened in both the positive and negative zones  
178 using NSM tape and EB carbon plate, respectively.

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185 **Table 1.** Specifications of the test specimens

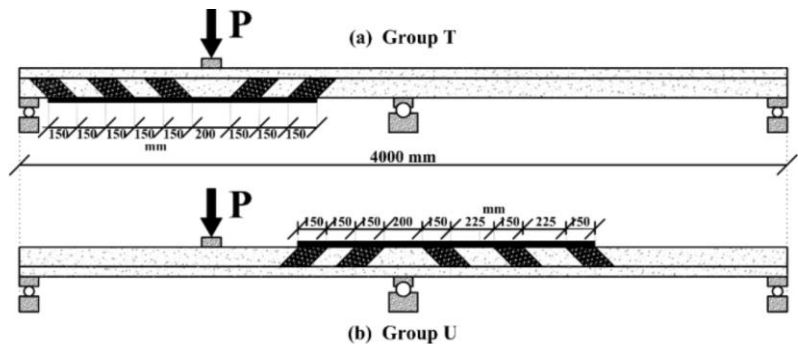
Beam	Positioning type	Strengthening configuration	Strengthening system	EA value of FRP (kN)	Anchorage system	$f_c^*$ (MPa)
T1	Normal	Control (no FRP)	N/A	-	N/A	35.6
T2	Normal	Positive zone	CFRP plate	9900	-	29.1
T3	Normal	Positive zone	CFRP plate	9900	U-wrap	36.1
T4	Normal	Negative zone	NSM CFRP tape	9100	-	27.3
T5	Normal	Both positive and negative zones	CFRP plate (Pos) NSM tape (Neg)	9900 9100	-	32.6
T6	Normal	Both positive and negative zones	CFRP plate (Pos) NSM tape (Neg)	9900 9100	U-wrap	35.3
U1	Upside-down	Control (no FRP)	N/A	-	N/A	34.7
U2	Upside-down	Positive zone	NSM CFRP tape	9100	-	35.7
U3	Upside-down	Negative zone	CFRP plate	9900	-	29.2
U4	Upside-down	Negative zones	CFRP plate	9900	U-wrap	32.5
U5	Upside-down	Both positive and negative zones	CFRP plate (pos) NSM tape (neg)	9900 9100	-	31.6
U6	Upside-down	Both positive and negative zones	CFRP plate (pos) NSM tape (neg)	9900 9100	U-wrap	30.3

186 Note:  $f_c^*$  = Average cylinder compressive strength of concrete on the day of testing.

187

188 **FRP anchorage system**

189 To anchor the EB FRP plates mechanically, U-wrap anchors were used in beams T3, T6, U4 and U6. The U-  
190 wraps were installed at an inclination of  $45^\circ$ , as research (Lee, 2010) has shown that the anchors are more  
191 effective in this direction than when vertical, to improve bond strength between the concrete and the FRP.  
192 Each anchor was made of carbon FRP sheet, and consisted of two similar pieces. Figs. 5(a) and (b) illustrate  
193 schematic images of the U-wraps installed on the soffit of the beams in group T, and over the central support  
194 in group U, respectively.



195 **Fig. 5.** The U-wrap anchors used for the FRP plates. (a) Group T; (b) Group U

The U-wraps were distributed along the entire length of the FRP plate, to help ensure the carbon plate would remain fully attached to the concrete along the full length during testing.

### Material properties

Each specimen was cast separately using a manual concrete mixer. The compressive strength of concrete was measured for each beam on the day of testing through crushing standard cylinders of 100 mm-diameter  $\times$  200-mm height. The measured values are summarized in Table 1. Also, properties of the steel reinforcements used are listed in Table 2 for the four different sizes of 3 mm, 6 mm, 8 mm and 12 mm.

**Table 2.** Mechanical properties of the steel reinforcement

Steel bar diameter (mm)	Yield strength, $f_y$ (MPa)	Ultimate strength, $f_u$ (MPa)	Young's modulus, $E_s$ (GPa)
3 (High yield smooth shear links)	710	768	213
6 (High yield ribbed shear links)	568	630	200
8 (High yield deformed bars)	575	633	200
12 (High yield deformed bars)	573	652	200

The CFRP material used for strengthening of the beams was a precured unidirectional plate of 1.4 mm thick  $\times$  50 mm wide. In addition, a precured carbon tape of cross-sectional area of 2 mm  $\times$  16 mm was adopted to strengthen the beams using the NSM method. The CFRP sheet used for the U-wraps was high-strength, unidirectional of 0.16 mm nominal thickness. The carbon sheet was applied to the beams by the wet-layup method, and impregnated in place using a two-part epoxy resin (Sikadur-330). The CFRP plate and tape were installed using a two-part epoxy structural adhesive (Sikadur-30). Average mechanical properties of the FRP materials measured through conducting unidirectional tensile testing on three samples, and of the epoxy resins provided by manufacturers, are listed in Table 3.

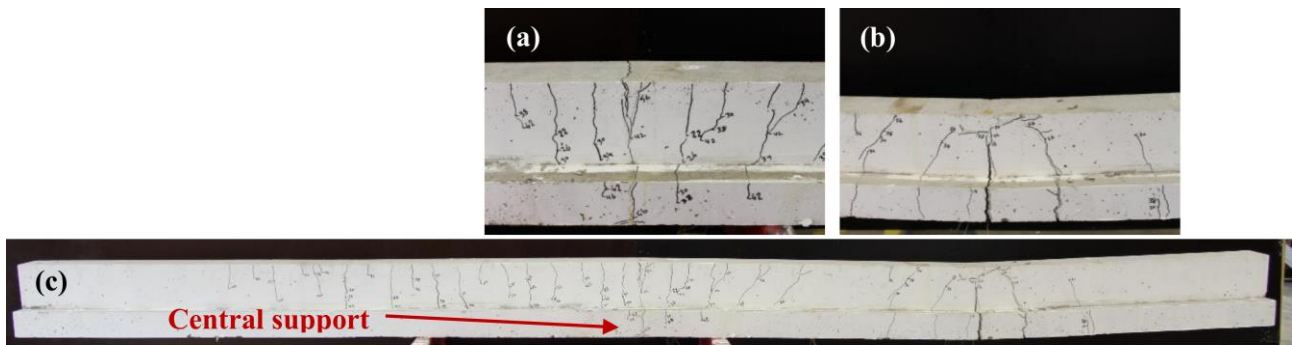
**Table 3.** Mechanical properties of the strengthening materials

Material	Ultimate tensile strength ( $\sigma_t$ )	Tensile modulus ( $E_t$ )	Ultimate strain ( $\epsilon_{tu}$ )	Bond strength
CFRP sheet	4230 MPa	238 GPa	1.78 %	N/A
CFRP plate	2590 MPa	145 GPa	1.79 %	N/A
CFRP tape	2410 MPa	141 GPa	1.68 %	N/A
Epoxy resin (Sikadur-330)	30 MPa	4.5 GPa	0.9 % (7 days)	> 4 MPa
Epoxy resin (Sikadur-30)	26-31 MPa	11.2 GPa	1.0 % (7 days)	> 4 MPa

## 215 Test results

### 216 Modes of failure

217 The unstrengthened control specimens, T1 and U1, failed in a conventional ductile manner, as expected for an  
 218 under-reinforced RC flexural member, through concrete crushing following yielding of the tension steel  
 219 reinforcement. Due to the loading arrangement adopted, the negative zone failed after initial plastic yielding  
 220 of the positive zone. Figs. 6(a) and (b) show respectively the positive and negative zones of beam U1 at ultimate  
 221 failure, when a plastic hinge has been formed in the negative zone following the earlier formation of a plastic  
 222 hinge in the positive zone. The major test results and findings are provided in Table 4 for all test specimens.



223 **Fig. 6.** Failure of beam U1: (a) Negative zone; (b) Positive zone; (c) Entire beam

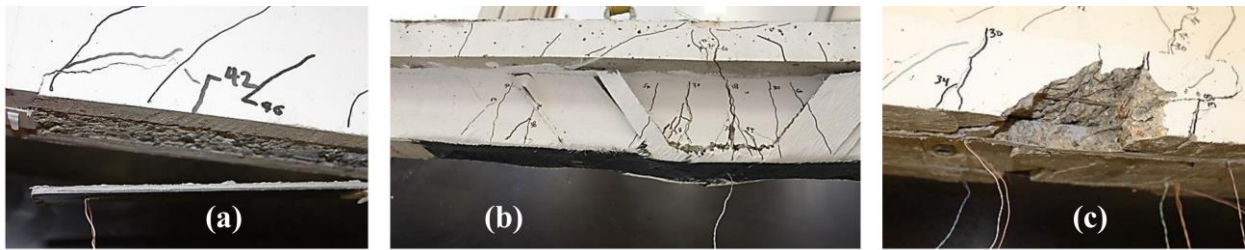
224 **Table 4.** Experimental results of the specimens

Beam	Failure mode	Anchorage	$P_{cr}$ (kN)	$P_{y-P}$ (kN)	$P_{y-N}$ (kN)	$P_u$ (kN)	$\epsilon_{deb}$	$P_R$ (kN)	MR (%)
T1	Concrete crushing	-	14	34	54	54	-	-	34
T2	FRP debonding	-	16	53	-	54	0.35 %	-	7
T3	FRP debonding	U-wrap	16	55	54	104	1.20 %	55	10
T4	FRP debonding	-	14	32	65	71	0.90 %	54	48
T5	FRP debonding	-	18	54*	65	53	0.35 %	71	9
T6	FRP debonding	U-wrap	18	63	-	114	1.20 %	55	13
U1	Concrete crushing	-	13	33	55	55	-	-	32
U2	FRP debonding	-	17	54	64	94	1.50 %	55	11
U3	FRP debonding	-	14	32	55*	62	0.35 %	56	42
U4	FRP debonding	U-wrap	14	33	66	72	0.90 %	55	52
U5	FRP debonding	-	19	58	83	92	1.00 %	85	18
U6	FRP debonding	U-wrap	20	61	83	106	1.40 %	83	20

225 Note:  $P_{cr}$  = Load at which first cracking occurred;  $P_u$  = Failure load (at FRP debonding);  
 226  $P_{y-P}$  = Yield load of the positive zone;  $P_{y-N}$  = Yield load of the negative zone;  
 227  $\epsilon_{deb}$  = Debonding strain of FRP; \*After FRP debonding  
 228  $P_R$  = Residual load capacity (indicating the ultimate load capacity after FRP debonding and before final concrete crushing);  
 229 MR = Experimental MR out of positive zone at failure  
 230

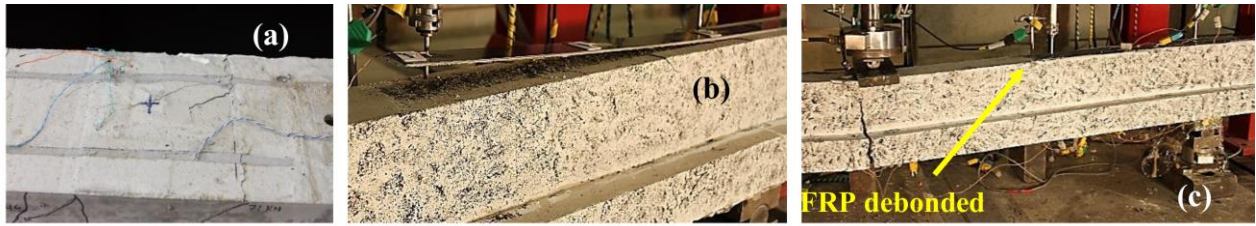
231 In all strengthened beams, FRP debonding occurred prior to any other form of failure, and signaled in every  
232 case the peak capacity. The tension reinforcement in the positive zone yielded prior to FRP debonding in all  
233 cases. As described in Table 4, the carbon plate debonded at applied loads of 54 kN and 104 kN in beams T2  
234 and T3, respectively. This demonstrated that the application of U-wraps was successful and effective such that  
235 the load resistance doubled, and the debonding strain was improved from 0.35% in beam T2 to 1.2% in beam  
236 T3.

237 The NSM tape in beam U2 debonded at an applied load of 94 kN. A large strain of 1.5% was recorded in the  
238 NSM tape at failure, demonstrating the effectiveness of using the NSM technique for strengthening of RC  
239 beams compared with other methods where ductility is required. The high bond strength between the concrete  
240 and FRP is obtained in the NSM technique due to the FRP being fully surrounded by epoxy resin. Debonding  
241 of the FRP in specimens T2, T3 and U2 is shown in Fig. 7.



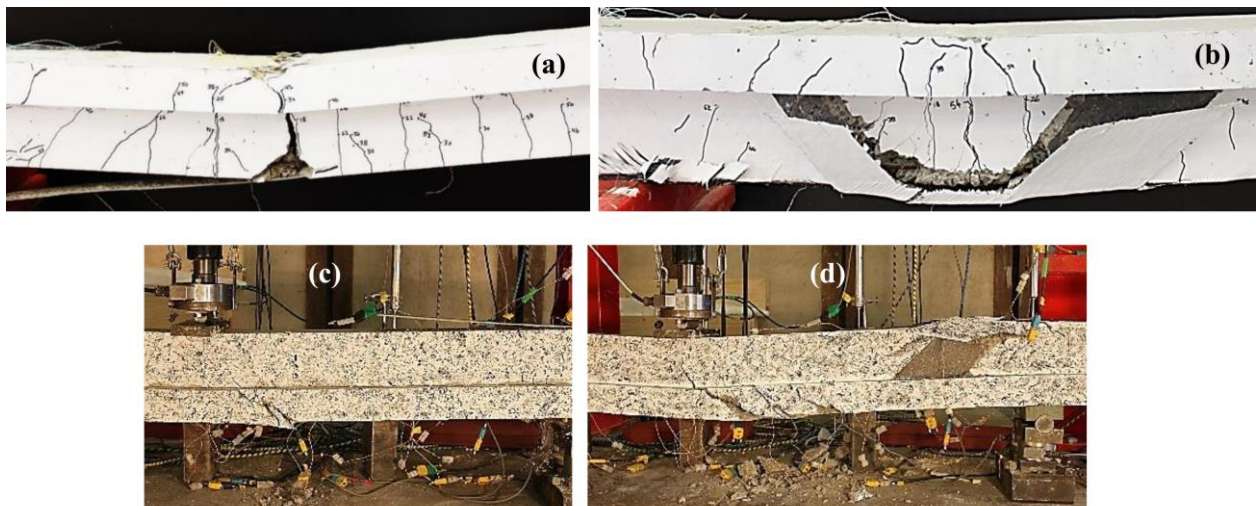
242 **Fig. 7.** Debonding of the FRP in the positive zone: (a) beam T2; (b) beam T3; (c) beam U2

243 Fig. 8 shows debonding of the FRP in beams T4, U3 and U4. The NSM tape in beam T4 debonded at an  
244 applied load of 71 kN. The strain recorded in the NSM tape at debonding was 0.9%, demonstrating a better  
245 bond performance between concrete and the FRP compared with that of EB FRP plates. The carbon plate  
246 debonded at applied loads of 62 kN and 72 kN in beams U3 and U4, respectively. The strains recorded in the  
247 FRP at debonding were 0.35% and 0.9% respectively, indicating the effectiveness of the U-wraps in  
248 postponing debonding, and improving the ductility of the strengthened section in beam U4, compared with  
249 that of beam U3.



**Fig. 8.** Failure of the FRP in the negative zone: (a) beam T4; (b) beam U3; (c) beam U4

The specimens strengthened in both the positive and negative zones (i.e. T5, T6, U5 and U6) exhibited a linear flexural behavior up to steel yield, and a partially ductile behavior after yielding of the steel reinforcement in the positive zone until FRP debonding. All four beams failed first in the positive zone through FRP debonding which occurred after steel yield. The negative zone failed later through the same failure mechanism. Fig. 9 depicts failure of the FRP in beams T5, T6, U5 and U6. The carbon plate in the positive zone debonded at applied loads of 53 kN and 114 kN in beams T5 and T6, respectively. The ultimate strains recorded in the FRP plate at debonding were 0.35% and 1.2% in the two beams, respectively. In beams U5 and U6, the NSM tape in the positive zone debonded at applied loads of 92 kN and 106 kN, respectively. Strains of 1% and 1.4% were recorded in the FRP tapes at debonding in the two beams. The load was further increased until the FRP plate in the negative zone debonded at 85 kN and 84 kN in beams U5 and U6, respectively. The debonding strain was 0.35% in beam U5, but it was 0.85% in beam U6 due to the application of U-wraps.



**Fig. 9.** Failure of the FRP in critical zones: (a) beam T5; (b) beam T6; (c) beam U5; (d) beam U6

As shown in Table 4, FRP strengthening of RC structures improves the load capacity of the structure provided that the FRP does not debond prematurely at a low strain. The effectiveness of strengthening was higher when the FRP was added to the positive zone. This was due to the loading arrangement adopted. The failure load



increase ratio ( $\lambda$ ) shows that the increase in load capacity could be from 67% to 111% when the positive zone was strengthened. As shown later in Figs. 10 and 11, ductility of the beams became higher when the negative zone only was strengthened.

### Load-Deflection response

Ductility of RC beams can be evaluated by measuring the deflection of critical points over the loading cycle (Mukhopadhyaya, 1998). Figs. 10 and 11 show the relationships between the applied load and mid-span deflection in the loaded span, recorded by LVDT2 (as depicted in Figs. 3 and 4) for the beams in group T and U, respectively. Three major phases are observed in the Load-Deflection relationships including the linear-elastic phase (from the beginning of loading to first concrete cracking), the cracked-elastic phase (from concrete cracking to yielding of the tension steel reinforcement), and the plastic phase (from steel yield to FRP debonding or concrete crushing).

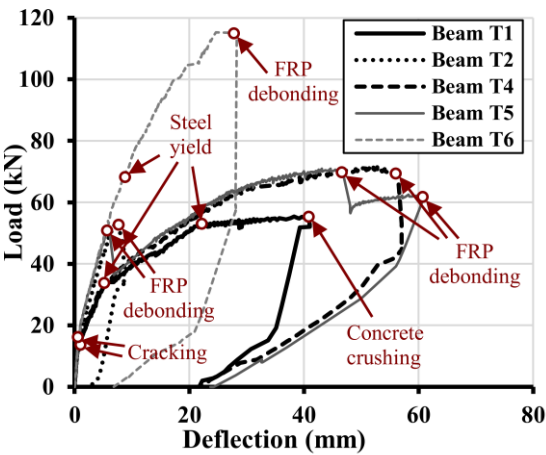


Fig. 10. Load-Deflection relationships for the beams in group T

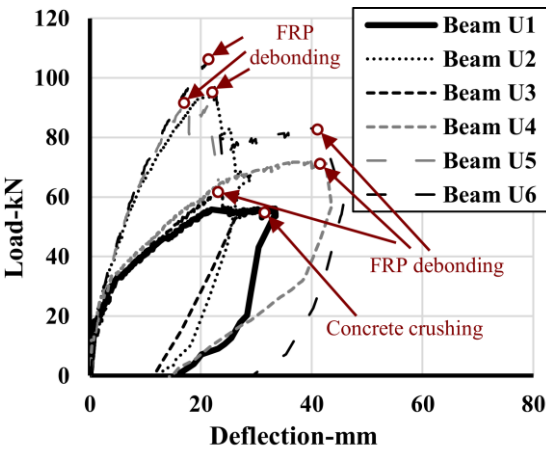
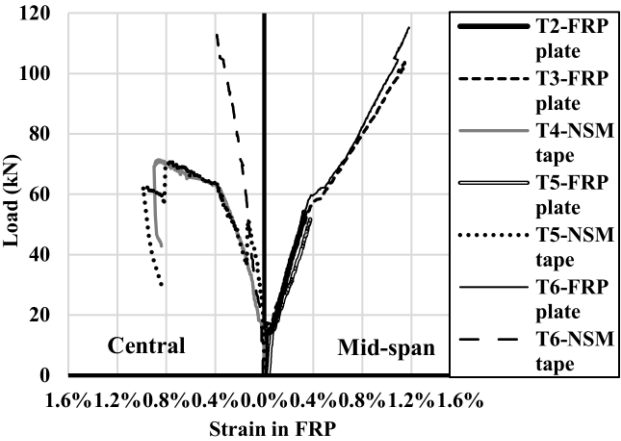


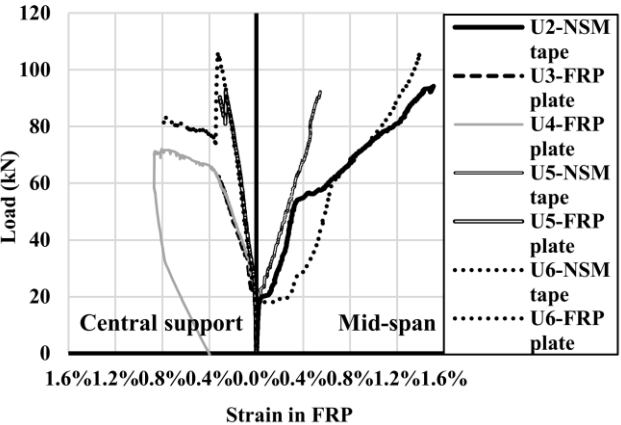
Fig. 11. Load-Deflection relationships for the beams in group U

279 **Load-Strain response**

280 Figs. 12 and 13 show the relationships between the applied load and strain in the FRP for the beams in groups  
281 T and U, respectively. Strain behavior of the unanchored FRP plate at the mid-span of beam T5 is similar to  
282 that of beam T2, and strain behavior of the anchored plate at the mid-span of beam T6 was similar to that of  
283 beam T3. After first concrete cracking, strain of the FRP increased considerably, and subsequently, the strain  
284 increased again after steel yield when a significant reduction in stiffness occurred. Comparison of the ultimate  
285 strain in the FRP plate between beams T2 and T3, between beams T5 and T6, between beams U3 and U4, and  
286 between beams U5 and U6 demonstrates the effectiveness of U-wrap anchors in improving the bond  
287 performance between concrete and the FRP plate.



288 **Fig. 12.** Load-Strain relationship in the FRP for the beams in group T



289 **Fig. 13.** Load-Strain relationship in the FRP for the beams in group U

290

291



292 ***Moment redistribution***

293 The bending moment redistributed out of the positive zone and into the negative zone at failure has been  
294 quantified at each load increment using Eq. (1). The values of experimental MR out of the positive zone at  
295 failure for all beams are listed in Table 4. The hypothetical elastic bending moment at failure ( $M_{elas}$ ) was  
296 calculated using elastic analysis, assuming no MR occurred and that the ultimate loading condition led to an  
297 entirely elastic distribution of bending moment.

298 The unstrengthened beams in both groups (T1 and U1) exhibited 34% and 32% MR at failure, respectively.  
299 As shown in Table 4 and later in Figs. 15 and 16, bending moment was redistributed without limit into FRP-  
300 strengthened zones during the experiments. The flexural behavior of the specimens strengthened only in the  
301 negative zone (i.e. T4, U3 and U4) was ductile (i.e. the internal steel yielded sufficiently) during loading  
302 despite being strengthened with FRP. This can be seen from Figs. 10 and 11, also from the percentage of MR  
303 at failure given in Table 4, compared with that of the control beams.

304 Although limited, it was found that bending moment was redistributed out of FRP-strengthened zones by up  
305 to 20% in these particular tests. The flexural behavior of the specimens strengthened only in the positive zone  
306 (i.e. T2, T3 and U2) was almost linear-elastic up to FRP failure. The ratio of positive moment to negative  
307 moment was destined to remain constant in the three beams where the FRP had been added to the positive  
308 zone in a large quantity so that the limits of bending strength in both zones were reached nearly simultaneously.  
309 This prevented significant MR from needing to occur. Adding FRP to the positive zone in beams T2 and U2  
310 caused a considerable reduction in the level of MR compared with that found in beams T1 and U1. In fact, the  
311 addition of elastic FRP resulted in 7%, 10% and 11% MR out of the strengthened zone in beams T2, T3 and  
312 U2 respectively.

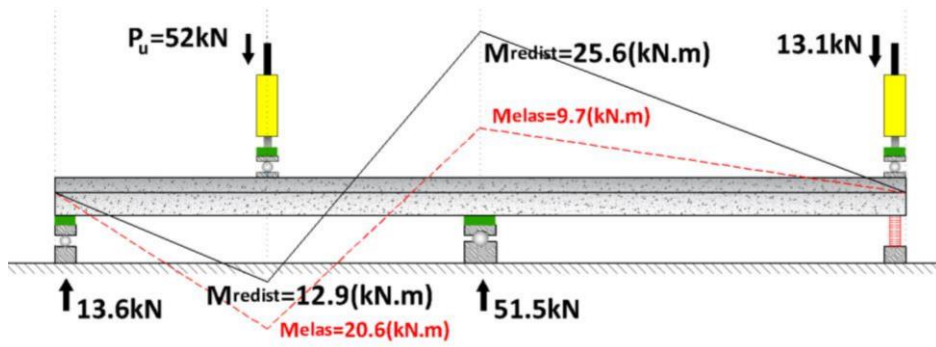
313 Overall, the beams strengthened in the positive zone (i.e. T2, T3, T5, T6, U2, U5 and U6) exhibited lower  
314 capacity for MR than the specimens strengthened only in the negative zone (i.e. T4, U3 and U4). The difference  
315 between amounts of MR in beams T2 and T3, in beams T5 and T6, and in beams U3 and U4 indicates the  
316 effectiveness of FRP anchorage. Moreover, the difference of MR between beams T2 and U2, and between T6  
317 and U6 can somewhat demonstrate the advantage of the NSM technique on the EB technique in improving

ductility. The experimental findings in general demonstrate that the amount of redistribution depends on the FRP stiffness (EA value) and quantity, method of installation, strengthening configuration, anchoring scheme and, of course, the precise geometry of the structure and loading conditions.

## **Analytical model**

The authors have previously proposed and developed a new analytical model to quantify MR in FRP-strengthened RC flexural members rigorously (Tajaddini et al., 2013; Tajaddini, 2015). The model is a novel theoretical strategy which employs basic structural mechanics to track MR, without any need for estimating rotation capacity or curvature ductility. Redistribution of bending moment in a beam is quantified through finding and updating the variation of flexural stiffness along the length of the beam over the loading cycle. Briefly, the model uses a numerical technique in which the beam is subdivided into a large number of vertical segments. Based on constitutive relationships, the Moment-Curvature plot is determined for each section using equilibrium of forces. Then, at each load increment, the flexural stiffness of each section along the beam is calculated from the Moment-Curvature relationship. An iterative approach is then used to find the actual distribution of bending moment along the beam, by including the effects of stiffness variation at each section and at each step. After each iteration, the flexural stiffness is updated for each section across the structure. The degree of MR can be determined at any point along the beam length, and at any stage of loading, until failure. The analytical model allows the flexural behavior of continuous FRP-strengthened RC beams, even if nonlinear, to be predicted. In addition, a wide variety of beam geometry, loading arrangement and strengthening technique or configuration can be considered. In this section, the analytical model is validated against the findings obtained from the experimental study on groups T and U.

The curvatures at the critical sections were calculated through the experimental data collected using the strain gauges installed on the steel reinforcement and FRP in the negative and positive zones. Fig. 14 illustrates a schematic image of the analytically predicted MR at failure in beam T1. The difference between the solid line (redistributed actual bending moment distribution at failure) and the dashed line (an elastic estimation of bending moment distribution at failure assuming no MR) shows graphically the degree of MR at failure along beam T1, based upon analytical results.



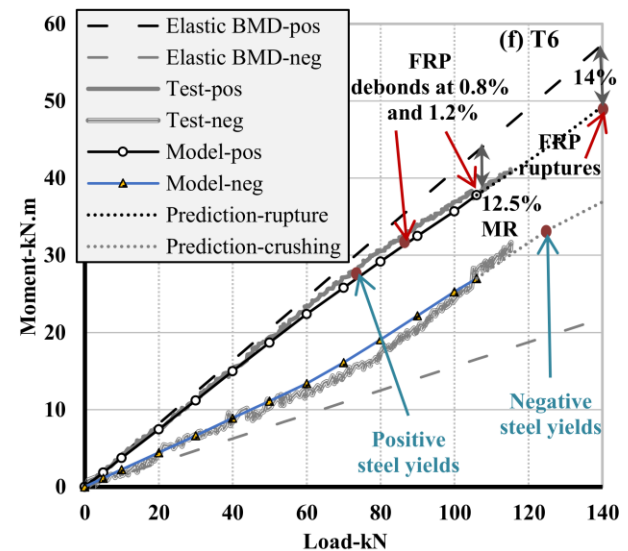
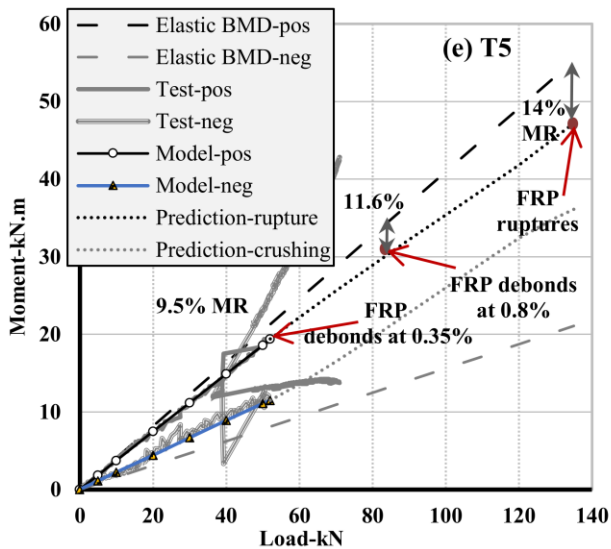
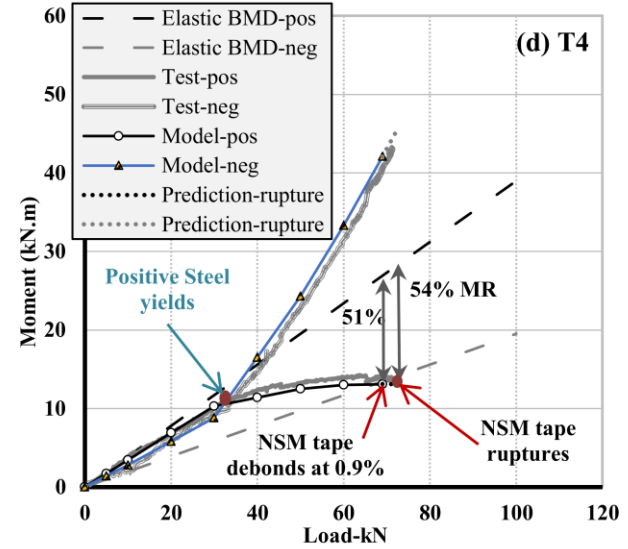
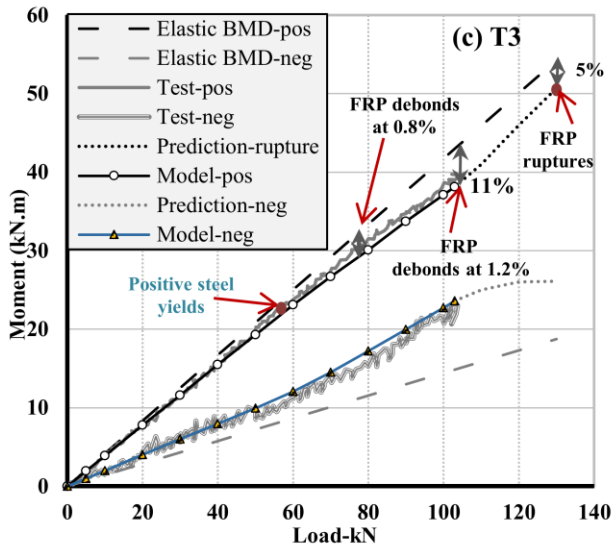
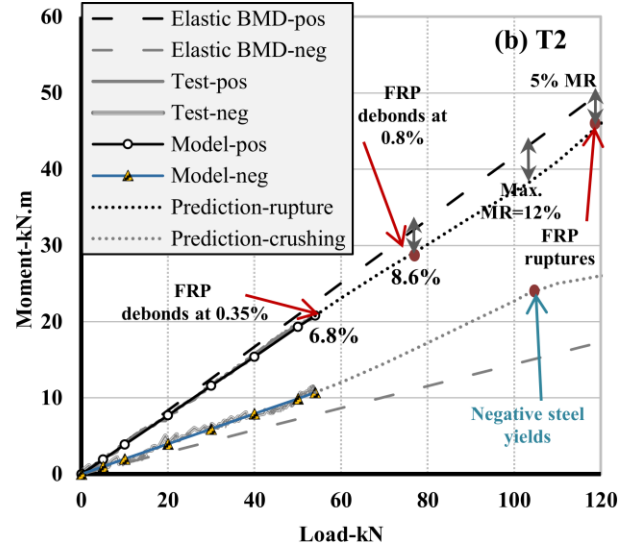
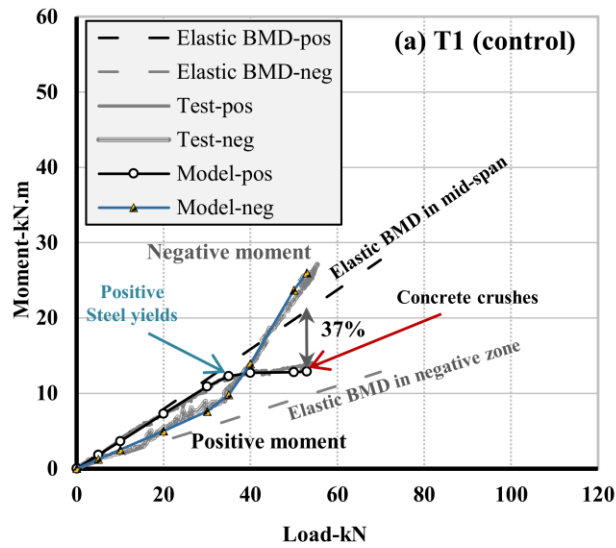
*Fig. 14. A schematic image of MR at failure in beam T1*

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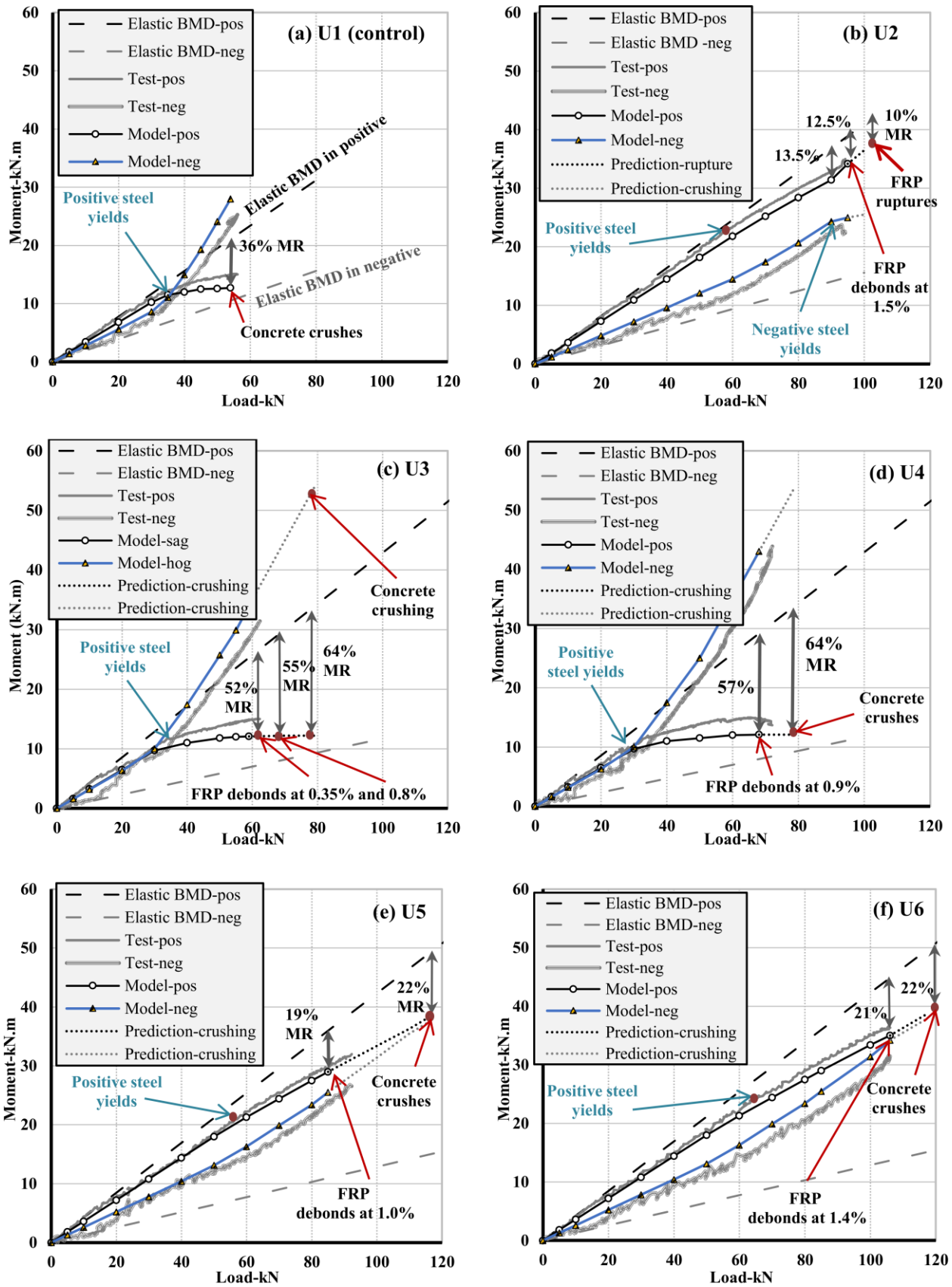
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Figs.15 and 16 illustrate the relationship between the applied load and bending moments in the positive and negative zones throughout loading for the beams in groups T and U respectively.



**Fig. 15.** Experimental Load-Moment curves vs analytical predictions for the beams in group T



*Fig. 16. Experimental Load-Moment curves vs analytical predictions for the beams in group U*

## Comparison and discussion

A good correlation can be seen between the experimental and analytical results for all tested beams in Figs. 15 and 16. This indicates how accurate the analytical model can predict the flexural behavior of FRP-strengthened RC structures from the beginning until failure. A brief comparison of the experimental and analytical results is provided in Table 5. The analytical predictions for MR in beams T1 and U1 were 37% and 36%, respectively. The analytical model shows predictions of 7% and 13% for MR in beams T2 and U2 at the same debonding strain recorded experimentally. If the FRP debonded at a typical strain of 0.8% (according to TR55, 2012), predictions for MR would be 9% and 8%, respectively.

**Table 5.** Summary of the result comparison for the tested beams

Beam	Strengthening location	Anchorage system	Experimental failure load (kN)	Analytical failure load (kN)	MR <sub>E</sub> (%)	MR <sub>An</sub> (%)	MR <sub>0.8</sub> (%)	MR <sub>max</sub> (%)
T1	N.A (control)	-	54	52	34	37	-	37
T2	Positive zone	-	54	54	7	7	9	12
T3	Positive zone	U-wrap	104	103	10	11	9	12
T4	Negative zone	-	71	69	48	51	50	54
T5	Both zones	-	52	52	9	10	12	14
T6	Both zones	U-wrap	114	106	13	12	11	14
U1	N.A (control)	-	55	54	32	36	-	36
U2	Positive zone	-	94	95	11	13	8	14
U3	Negative zone	-	61	59	42	52	55	64
U4	Negative zone	U-wrap	72	68	52	57	55	64
U5	Both zones	-	92	85	18	19	13	22
U6	Both zones	U-wrap	106	106	20	21	13	22

Note: MR<sub>E</sub> = Experimentally recorded MR out of the positive zone;

MR<sub>An</sub> = Analytical prediction for MR out of positive zone at experimentally recorded debonding strain;

MR<sub>0.8</sub> = Analytical prediction for MR out of positive zone at a typical debonding strain of 0.8%;

MR<sub>max</sub> = Maximum possible capacity for redistribution provided that either the FRP ruptures or the concrete crushes;

Anchoring the FRP plate increased the MR from 7% in beam T2 to 10% in beam T3. As shown in Fig. 15(c), the analytical model predicts a failure load of 130 kN, instead of 104 kN recorded experimentally, assuming that the FRP would fail through rupture at its full strain capacity, not debonding. In this case, 5% MR out of the strengthened zone would occur at failure, but the maximum possible capacity for MR would be 11% which occurs at 107 kN at which point the steel reinforcement would yield in the negative zone, and the level of MR out of the positive zone would subsequently be less. Comparison of the results in beams T2, T3 and U2

369 demonstrates that the NSM technique provides a higher potential for MR than the EB technique even if the EB  
370 FRP plate is anchored.

371 Although it is not significant in the elastic range, MR is initiated after first concrete cracking due to non-  
372 uniform stiffness along the length of the beam, and intensifies usually after steel yielding. Beam T4 exhibited  
373 48% MR out of the positive zone which was more than that of the control beam. This demonstrated that adding  
374 FRP can even improve the overall ductility of an RC member, provided that a suitable strengthening  
375 configuration is adopted. It is predicted that the full capacity for MR in beam T4 would be 54% if the FRP  
376 failed through rupture instead of debonding. MRs were 42% and 52% in beams U3 and U4 at failure,  
377 respectively. This again indicates the influence of anchoring of the FRP on the level of MR. It is predicted by  
378 the model that the full capacity for MR would be 64% in the two beams, which seems rather promising for an  
379 FRP-strengthened RC beam.

380 As can be observed in Table 5, if both critical zones are strengthened, more bending moment can be  
381 redistributed compared with the beams strengthened only in the positive zone. However, it is recommended  
382 (Denton, 2007) not to strengthen both positive and negative zones together in reality, this configuration was  
383 considered here for completeness. Accordingly, beam T5 exhibited 9% MR out of the positive zone at failure,  
384 and beam T6 showed 13% MR due to anchoring the FRP plate. It is predicted by the analytical model that the  
385 maximum capacity for MR in beams T5 and T6 would be 14%, if the EB FRP plate could reach its full strain  
386 capacity of 1.79% before failure.

387 MR of 18% and 20% occurred out of the NSM-strengthened positive zone in beams U5 and U6, respectively.  
388 These significant amounts of redistribution demonstrate that MR can be feasible out of FRP-strengthened  
389 zones if an appropriate quantity of FRP is used, and premature debonding of the FRP is prevented. Prediction  
390 for full capacity of MR, if debonding can be prevented, is 22% for beams U5 and U6, in which failure would  
391 occur through concrete crushing prior to FRP rupture (as shown in Fig. 16(e) and 16(f)). It should be noted  
392 that, as the results show, if significant MR is to occur, it is necessary that the internal steel reinforcement  
393 should yield, which in turn causes a considerable increase in the curvature of the critical section.

394

## 395   **Conclusions**

396   Redistribution of bending moments in FRP-strengthened continuous RC flexural members has been addressed  
397   and investigated in this paper both experimentally and analytically. Twelve large-scale concrete T-beams were  
398   tested, and an analytical strategy for the quantification of MR was described. The following conclusions are  
399   drawn based on the experimental and analytical findings:

- 400       • The experimental findings of the current research indicate that an FRP-strengthened zone in an RC  
401       member can redistribute bending moment significantly. Up to 20% MR out of strengthened zones was  
402       found here. However, this is highly dependent on the initial conditions of the member before  
403       strengthening, FRP quantity, configuration and technique of strengthening, and anchoring scheme.
- 404       • The new analytical model described here can reasonably model the flexural behavior of FRP-  
405       strengthened RC structures such that MR is quantified, at any stage of loading up to failure, at  
406       reasonable accuracy.
- 407       • Both analytical and experimental results indicate that if only the zone into which bending moment is  
408       redistributed is strengthened, the degree of MR in this beam will be higher than that possible in the  
409       original unstrengthened beam. This is because the zone from which MR initiates is unstrengthened  
410       and ductile, while the strengthened zone has a higher strength compared with that before strengthening.  
411       This allows more bending moment to be redistributed into this zone. This is valid even if the FRP  
412       debonds at a low strain. Thus, MR into FRP-strengthened zones should be allowed without undue  
413       limitations, whereas current design guides and codes can presently be rather conservative in handling  
414       this issue.
- 415       • If a concrete beam has sufficient capacity originally for MR, the possibility for considerable  
416       redistribution of bending moment should not be ignored after FRP strengthening, even out of the  
417       strengthened zones. However, if adding FRP causes the ratio of positive bending moment to negative  
418       bending moment to be more or less constant over the loading cycle, no (or negligible) MR will be  
419       possible.



- An appropriate mechanical anchoring of the FRP can significantly improve ductility of the retrofitted section and, as a result, the degree of MR can increase. The inclined U-wrap anchors exhibited high effectiveness in anchoring the externally-bonded FRP strengthening materials, such that the ultimate strain in the FRP could increase from 0.35% to 1.2% in most cases, which caused up to 10% increase in MR compared with that of the unanchored beam.
- The near surface mounted (NSM) FRP strengthening technique exhibited a more effective structural performance than the externally-bonded FRP plate strengthening technique, with the ultimate strain being larger in the NSM FRP than in the plated FRP. This better performance of the NSM technique was valid even when the FRP plate was anchored mechanically. Hence, it is recommended that the NSM technique is considered when MR is desirable attribute during design.
- The experimental and analytical findings indicate that strengthening of only the zone into which MR occurred was most effective compared with strengthening of both negative and positive zones together in terms of MR. The case when only the zone from which bending moment is redistributed was strengthened was least effective. Failure was catastrophic in the case when both critical zones were strengthened together, compared with that of single-zone strengthening only, such that no residual capacity was observed in the beam after failure of the FRP. In fact, it was observed that the second-critical-zone FRP debonded suddenly and catastrophically, immediately after the first-critical-zone FRP debonded. For this reason, it is recommended that continuous structures are strengthened preferably only in the zones into which MR will occur, and that such redistribution is exploited.

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## 446    **Notations**

447    The following symbols are used in this paper:

448	$E_s$ = Young's modulus of steel	465	$MR_{0.8}$ = Analytical prediction for moment
449	$E_f$ = Tensile modulus of the FRP	466	redistribution at strain of 0.8%
450	$EA$ = Tension stiffness of the FRP	467	$MR_{max}$ = Maximum capacity for moment redistribution
451	$EI$ = Flexural stiffness	468	if debonding is prevented
452	$f_c$ = Cylinder compressive strength of concrete	469	$P$ = Applied load
453	$f_y$ = Yield strength of steel reinforcement	470	$P_{cr}$ = Load at concrete cracking
454	$f_u$ = Ultimate strength of steel	471	$P_{y-P}$ = Yield load of positive zone
455	$M_{elas}$ = Theoretical bending moment determined from	472	$P_{y-N}$ = Yield load of negative zone
456	elastic analysis	473	$P_R$ = Residual load capacity
457	$M_{redis}$ = Redistributed bending moment	474	$P_u$ = Ultimate (failure) load
458	$M_{cr}$ = Bending moment at cracking	475	$\varepsilon_{fu}$ = Ultimate strain
459	$M_u$ = Moment capacity	476	$\varepsilon_{deb}$ = Debonding strain
460	$MR$ = Moment redistribution	477	$\sigma_f$ = Ultimate tensile strength
461	$MR_E$ = Experimental moment redistribution out of	478	$\varphi_u$ = Ultimate curvature
462	positive zone	479	$\varphi_y$ = Curvature at steel yield
463	$MR_{AN}$ = Analytical prediction for moment		
464	redistribution at experimental strain		

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555    **List of figures:**

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